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Finite element analysis for co-current and counter-current parallel flow three-fluid heat exchanger

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Abstract

Purpose – To study the thermal performance of both co-current and counter-current parallel flow heat exchangers. The hot stream is assumed to flow in the middle of two cold streams and exchange heat with them.

Design/methodology/approach – The dimensionless governing equations are derived based on the conservation of energy principle and solved using FEM based on subdomain collocation method and Galerkin's method. The results show that the subdomain collocation method is more accurate than the Galerkin's method, as observed when the results obtained are compared with the analytical results for the classical two-fluid heat exchangers.

Findings – The results are presented in terms of effectiveness and number of transfer units (Ntu) for different values of the governing parameters. The governing parameters are the Ntu, the heat capacity ratios, the overall heat transfer coefficient ratio, and the inlet temperatures parameter. The results show that the effectiveness of the three-fluid heat exchanger is always higher than that of classical two-fluid flow heat exchanger for fixed values of the governing parameters. The results also show that for fixed values of the governing parameters, the effectiveness of the counter-current is higher than the co-current parallel flow three-fluid heat exchangers.

Research limitations/implications – One-dimensional governing equations are derived based on the conservation of energy principle. The ranges of the governing parameters are: Ntu (0:5), the heat capacity ratios (0:1,000), the overall heat transfer coefficient ratio (0:2), and the inlet temperatures parameter (0:1).

Practical implications – Both co-current and counter-current parallel flow heat exchangers are used in the thermal engineering applications. The design and performance analysis of these heat exchangers are of practical importance.

Originality/value – This paper provides the details of the performance analysis of co-current and counter-current parallel flow heat exchangers, which can be used in thermal design.

Keywords Heat exchangers, Fluid mechanics, Finite element analysis

Paper type Research paper

Nomenclature

C_p = specific heat
H = heat transfer coefficient ratio
L = heat exchanger length
$\dot{m}$ = mass flow rate
Ntu = number of transfer unit
P = contact perimeter
q_{max} = maximum possible heat transfer rate
R = heat capacity ratio

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T = temperature
U = overall heat transfer coefficient
W = weighted parameter
x = coordinate along heat exchanger
X = dimensionless x

Greek symbols
ε = effectiveness
θ = dimensionless temperature

Subscripts
c1 = cold fluid 1
c2 = cold fluid 2
e = element
h = hot fluid
i = inlet
o = outlet
1 = between hot fluid and cold fluid 1
2 = between hot fluid and cold fluid 2

1. Introduction

Owing to many engineering applications of heat exchangers, intensive research has been carried out for the last several decades. Many research efforts addressed the enhancement of heat transfer between two or more fluids with different temperatures through special flow configuration and shape of the exchanger contact surface area.

Although heat exchanger designs have shown extensive progress, they are generally limited to few of many possible flow arrangements and mostly restricted to two fluid heat exchangers. Recently, the literature shows fast progress studies on some designs that involve three or multi-fluid heat exchangers (Chato *et al.*, 1971; Sekulic, 1994; Lalvani *et al.*, 2000; Luo *et al.*, 2002). The engineering applications of the multi-fluid heat exchangers include the petro-chemical, aerospace, separation of air, helium-air separation, purification and liquefaction of hydrogen, etc. Many micro-scale heat exchangers with two working fluids can be treated as three-fluid heat exchangers where the third fluid is the ambient with infinite thermal capacity.

Sekulic and Shah (1995) have presented in detail a review on thermal design theory of three-fluid heat exchangers, where they have allowed the third fluid temperature to vary according to the prevailing thermal communications while neglecting interaction with the ambient.

Recently, Shrivastava and Ameel (2004a) have developed a mathematical model based on Sekulic and Shah (1995)) review for three-fluid heat exchanger. In these studies (Shrivastava and Ameel, 2004a), six non-dimensional design parameters were identified and their effect on the temperature distributions of the different fluid streams were presented. Several effectiveness definitions have been proposed to assess the performance of three-fluid heat exchangers. Shrivastava and Ameel (2004b) have defined six different effectiveness parameters based on the five identified engineering goals of three-fluid heat exchangers, which are:

(1) heating the cold fluid;
(2) cooling the hot fluid;
(3) cooling the intermediate fluid;
(4) heating the intermediate fluid; and
(5) maximizing the enthalpy change of the central fluid stream or the lateral fluid streams.

The comparison of longitudinal wall conduction effect on the cross flow heat exchangers including three-fluid streams with different arrangements were carried out by Yuan and Kou (2001). The performance of the three-fluid cross flow heat exchangers was studied also by Yuan (2003) with the effect of the inlet flow maldistribution.

Seetharamu *et al.* (2004) have analyzed a three fluid heat exchanger for parallel flow situation. They have considered a double pipe heat exchanger with heat losses to ambient and compared with available literature. They have also demonstrated that the methodology can be applied to Buoyonet heat exchanger. Barron (1984) has developed a mathematical model for cryogenic heat transfer where one of the fluids in a two-fluid heat exchanger is interacting with the ambient.

It is noticeable from the literature that attention has been given to mathematical modeling of the three-fluid heat exchangers, with main focus to find the thermal fields in the heat exchangers using analytical (Sekulic and Shah [6], Shrivastava and Ameel [8]), semi-analytical methods (Luo *et al.*, 2003; Bielski and Malinowski, 2003) as well as numerical methods (Yuan and Kou, 2001; Yuan, 2003; Seetharamu *et al.*, 2004).

In the present study, the finite element method is used to study the thermal analysis for the three-fluid parallel and counter flow heat exchanger. The schematic diagram of the parallel flow and counter flow three-fluid heat exchanger are shown in Figure 1 in which the hot fluid is flowing in between the cold fluids. The study includes the effect of the overall heat transfer coefficients between the hot fluid and the two cold fluids, which are assumed to be different, and the effect of the heat capacity of all the three fluids. The study includes also the effect of the contact perimeter of the two cold channels which are not necessarily be same.

2. Governing equations

Consider the steady-state flow of the hot fluid in the middle channel between two steady-state parallel flow or two counter flow cold fluids as shown in Figure 1(a) and (b), respectively. The boundary conditions are given by specifying the inlet temperatures of the three streams. To generalize the formulations, it is assumed that the over all heat transfer coefficient U between the hot fluid and the two cold fluids are different. Variable contact areas between the hot channel and the two cold fluids are assumed. Applying the conservation of energy principle for each of the three streams, the following equations are derived:

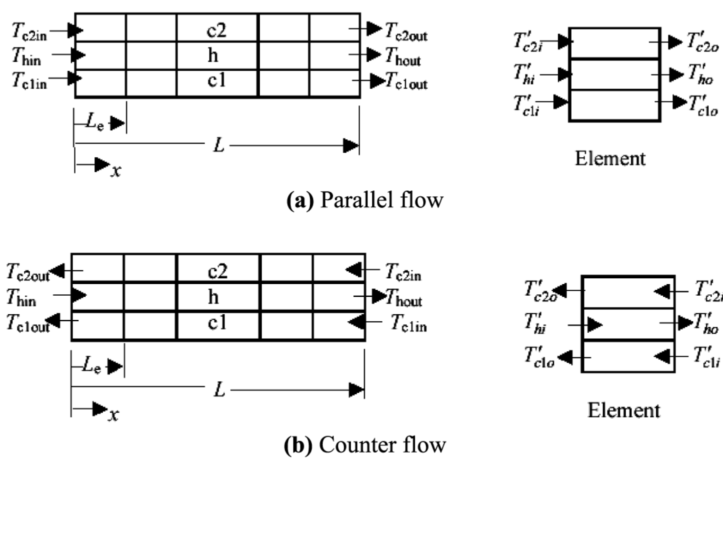

Figure 1. Schematic diagram for three-fluid heat exchangers

$$(\dot{m}c_{\mathrm{p}})_{\mathrm{c1}}\frac{\mathrm{d}T_{\mathrm{c1}}}{\mathrm{d}x} = \pm U_1P_1(T_{\mathrm{h}} - T_{\mathrm{c1}}) \tag{1}$$

$$(\dot{m}c_{\mathrm{p}})_{\mathrm{h}}\frac{\mathrm{d}T_{\mathrm{h}}}{\mathrm{d}x} = -U_1P_1(T_{\mathrm{h}} - T_{\mathrm{c1}}) - U_2P_2(T_{\mathrm{h}} - T_{\mathrm{c2}}) \tag{2}$$

$$(\dot{m}c_{\mathrm{p}})_{\mathrm{c2}}\frac{\mathrm{d}T_{\mathrm{c2}}}{\mathrm{d}x} = \pm U_2P_2(T_{\mathrm{h}} - T_{\mathrm{c2}}) \tag{3}$$

where the positive sign in equations (1) and (3) is for parallel flow and the negative sign is for the counter flow, U_1 and U_2 are the over all heat transfer coefficient between the hot fluid and c1 cold fluid and c2 cold fluid, respectively. P_1 and P_2 are the contact perimeter with the hot channel of c1 channel and c2 channel, respectively.

In order to simplify and generalize the equations, the following dimensionless variables are introduced:

$$\theta = \frac{T - T_{\mathrm{c1in}}}{T_{\mathrm{hin}} - T_{\mathrm{c1in}}}, \quad \text{and} \quad X = \frac{x}{L_{\mathrm{e}}} \tag{4}$$

where, L_{e} is the length of the element (Figure 1) and the inlet temperatures of the two cold streams may not be equal. It can be shown that the dimensionless forms of the governing equations are:

$$\frac{\mathrm{d}\theta_{\mathrm{c1}}}{\mathrm{d}X} = \frac{\pm U_1P_1L_{\mathrm{e}}}{(\dot{m}c_{\mathrm{p}})_{\mathrm{c1}}}(\theta_{\mathrm{h}} - \theta_{\mathrm{c1}}) \tag{5}$$

$$\frac{\mathrm{d}\theta_{\mathrm{h}}}{\mathrm{d}X} = \frac{-U_1P_1L_{\mathrm{e}}}{(\dot{m}c_{\mathrm{p}})_{\mathrm{h}}}(\theta_{\mathrm{h}} - \theta_{\mathrm{c1}}) - \frac{U_2P_2L_{\mathrm{e}}}{(\dot{m}c_{\mathrm{p}})_{\mathrm{h}}}(\theta_{\mathrm{h}} - \theta_{\mathrm{c2}}) \tag{6}$$

$$\frac{\mathrm{d}\theta_{\mathrm{c2}}}{\mathrm{d}X} = \frac{\pm U_2P_2L_{\mathrm{e}}}{(\dot{m}c_{\mathrm{p}})_{\mathrm{c2}}}(\theta_{\mathrm{h}} - \theta_{\mathrm{c2}}) \tag{7}$$

The governing parameters can be combined together in order to reduce the number of the parameters as follows:

$$\mathrm{Ntu}_{\mathrm{e}} = \frac{U_1P_1L_{\mathrm{e}}}{(\dot{m}c_{\mathrm{p}})_{\mathrm{h}}};\quad R_1 = \frac{(\dot{m}c_{\mathrm{p}})_{\mathrm{h}}}{(\dot{m}c_{\mathrm{p}})_{\mathrm{c1}}};\quad R_2 = \frac{(\dot{m}c_{\mathrm{p}})_{\mathrm{h}}}{(\dot{m}c_{\mathrm{p}})_{\mathrm{c2}}};\quad H = \frac{U_2P_2}{U_1P_1};\quad \Theta = \frac{T_{\mathrm{c2in}} - T_{\mathrm{c1in}}}{T_{\mathrm{hin}} - T_{\mathrm{c1in}}} \tag{8}$$

Therefore, the final forms of the dimensionless governing equations are:

$$\frac{\mathrm{d}\theta_{\mathrm{c1}}}{\mathrm{d}X} = \pm \mathrm{Ntu}_{\mathrm{e}}R_1(\theta_{\mathrm{h}} - \theta_{\mathrm{c1}}) \tag{9}$$

$$\frac{\mathrm{d}\theta_{\mathrm{h}}}{\mathrm{d}X} = -\mathrm{Ntu}_{\mathrm{e}}(\theta_{\mathrm{h}} - \theta_{\mathrm{c1}}) - \mathrm{Ntu}_{\mathrm{e}}H(\theta_{\mathrm{h}} - \theta_{\mathrm{c2}}) \tag{10}$$

$$\frac{\mathrm{d}\theta_{\mathrm{c2}}}{\mathrm{d}X} = \pm \mathrm{Ntu}_{\mathrm{e}}HR_2(\theta_{\mathrm{h}} - \theta_{\mathrm{c2}}) \tag{11}$$

Equations (9)-(11) should be solved with the specified inlet temperatures to find the outlet temperatures from each channel. Therefore, the dimensionless boundary conditions can be defined (equation (4)) as: $\theta_{\mathrm{hin}} = 1$, $\theta_{\mathrm{c1in}} = 1$ and $\theta_{\mathrm{c2in}} = \Theta$.

3. Finite element method

The heat exchanger is descretized into a number of elements as shown schematically in Figure 1. Using the method of minimizing the weighted residual (Lewis *et al.*, 2004) to solve equations (9)-(11) as:

$$\int_0^1 W\left\{\frac{\mathrm{d}\theta_{\mathrm{c1}}}{\mathrm{d}X} \mp \mathrm{Ntu_e} R_1(\theta_{\mathrm{h}} - \theta_{\mathrm{c1}})\right\}\mathrm{d}X = 0 \tag{12}$$

$$\int_0^1 W\left\{\frac{\mathrm{d}\theta_{\mathrm{h}}}{\mathrm{d}X} + \mathrm{Ntu_e}(\theta_{\mathrm{h}} - \theta_{\mathrm{c1}}) + \mathrm{Ntu_e} H(\theta_{\mathrm{h}} - \theta_{\mathrm{c2}})\right\}\mathrm{d}X = 0 \tag{13}$$

$$\int_0^1 W\left\{\frac{\mathrm{d}\theta_{\mathrm{c2}}}{\mathrm{d}X} \mp \mathrm{Ntu_e} H R_2(\theta_{\mathrm{h}} - \theta_{\mathrm{c2}})\right\}\mathrm{d}X = 0 \tag{14}$$

Assuming a linear variation of the hot and cold fluids in a single element for the parallel flow as:

$$\theta'_{\mathrm{h}} = N_1\theta'_{\mathrm{hi}} + N_2\theta'_{\mathrm{ho}} \tag{15a}$$

$$\theta'_{\mathrm{c1}} = N_1\theta'_{\mathrm{c1i}} + N_2\theta'_{\mathrm{c1o}} \tag{15b}$$

$$\theta'_{\mathrm{c2}} = N_1\theta'_{\mathrm{c2i}} + N_2\theta'_{\mathrm{c2o}} \tag{15c}$$

and for the counter flow as:

$$\theta'_{\mathrm{h}} = N_1\theta'_{\mathrm{hi}} + N_2\theta'_{\mathrm{ho}} \tag{15d}$$

$$\theta'_{\mathrm{c1}} = N_2\theta'_{\mathrm{c1i}} + N_1\theta'_{\mathrm{c1o}} \tag{15e}$$

$$\theta'_{\mathrm{c2}} = N_2\theta'_{\mathrm{c2i}} + N_1\theta'_{\mathrm{c2o}} \tag{15f}$$

Where the prime denotes the element temperatures and the shape functions N_1 and N_2 are given by $N_1 = 1 - X$ and $N_2 = X$. Substitution of these approximations in equations (12)-(14), the set of three algebraic equations can be obtained if the weighted parameter W is defined.

In the present analysis, two methods are used and their results are compared in the next section. The first method is the subdomain collocation in which the weights are taken to be unity and the second method is the Galerkin's method in which the weights are taken to be shape functions N_1 and N_2. Then three algebraic equations can be obtained for the Subdomain collocation method. Three more equations can be formed from the inlet boundary conditions. If the number of elements is more than one, care should be taken in assigning correct boundary conditions, where in these cases the outlet temperature from one element is equal to the inlet temperature to the next element in the direction of the fluid flow. Therefore, the discretized governing equations can be written in matrix form for each element as:

$$[K]\{\theta'\} = \{f\} \tag{16}$$

where $[K]$ is known as the stiffness matrix and it is (6×6) matrix for each element, the details of the matrices are given in the Appendix A for both parallel and counter flow arrangements using the subdomain collocation method. Assembling the element matrix form of the governing equations for all the elements in the solution domain leads to the global matrix form of the governing equations in the whole solution domain. The resultant global matrix form is solved by Gauss-Jordan elimination method for the dimensionless temperatures along the heat exchanger.

4. Results and discussion

The effectiveness – number of transfer units (ε-Ntu) method is regarded as powerful tool and easy to implement in both design and performance calculations of the heat exchangers. Therefore, this method is used to present the results of the present study.

As mentioned earlier, several effectiveness definitions have been proposed to assess the performance of three-fluid heat exchangers. In the present study for three-fluid heat exchanger, the effectiveness can be defined, as usual, as the ratio of the actual heat transfer rate to the maximum possible heat transfer rate. The actual heat transfer rate is:

$$q_{\mathrm{act}} = (\dot{m}c_{\mathrm{p}})_{\mathrm{h}}(T_{\mathrm{hin}} - T_{\mathrm{hout}}) \tag{17a}$$

The maximum possible heat transfer rate can be calculated as:

When

$$(\dot{m}c_{\mathrm{p}})_{\mathrm{h}} < \{(\dot{m}c_{\mathrm{p}})_{\mathrm{c1}} + (\dot{m}c_{\mathrm{p}})_{\mathrm{c2}}\}$$

$$q_{\max} = (\dot{m}c_{\mathrm{p}})_{\mathrm{h}}(T_{\mathrm{hin}} - \mathrm{Min}[T_{\mathrm{c1in}}, T_{\mathrm{c2in}}]) \tag{17b}$$

otherwise

$$q_{\max} = (\dot{m}c_{\mathrm{p}})_{\mathrm{c1}}(T_{\mathrm{hin}} - \mathrm{Min}[T_{\mathrm{c1in}}, T_{\mathrm{c2in}}]) + (\dot{m}c_{\mathrm{p}})_{\mathrm{c2}}(T_{\mathrm{hin}} - \mathrm{Min}[T_{\mathrm{c1in}}, T_{\mathrm{c2in}}]) \tag{17c}$$

Therefore, the effectiveness can be defined in terms of the dimensionless variables as:

$$\varepsilon = \frac{\theta_{\mathrm{hin}} - \theta_{\mathrm{hout}}}{\mathrm{Min}\left\{1, \left(R_1^{-1} + R_2^{-1}\right)\right\}\left(\theta_{\mathrm{hin}} - \mathrm{Min}[\theta_{\mathrm{c1in}}, \theta_{\mathrm{c2in}}]\right)} \tag{18}$$

It is noted that this definition is applicable for the classical two-fluid heat exchanger with $R_2^{-1} = 0$ as well as three-fluid counter flow heat exchangers.

The numerical scheme is tested for the analysis of the classical parallel flow and counter flow two-fluid heat exchanger by setting $H = 0$ in the present formulation. The results of both the Subdomain collocation method and Galerkin's methods for the classical two-fluid parallel flow and counter flow heat exchanger are listed in Tables I to IV using different number of elements together with the following well-known analytical formulas (Incropera and DeWitt (2002):

Table I. Comparison of the values of the effectiveness of the classical parallel-flow two-fluid heat exchanger ($H = 0$) using different number of elements (Subdomain collocation method) for $R_1 = 0.5$

Ntu	Equation (19)	2 elements	ε 4 elements	8 elements	16 elements
0.5	0.3518	0.3546	0.3525	0.3519	0.3518
1.0	0.5179	0.5289	0.5206	0.5186	0.5181
1.5	0.5964	0.6144	0.6006	0.5974	0.5967
3.0	0.6593	0.6644	0.6626	0.6601	0.6595
5.0	0.6663	0.6049	0.6667	0.6665	0.6663

For two-fluid parallel flow:

$$\varepsilon = \frac{1 - \exp\{-\mathrm{Ntu}(1 + R_{\min})\}}{1 + R_{\min}} \tag{19}$$

For two-fluid counter flow:

$$\varepsilon = \frac{1 - \exp\{-\mathrm{Ntu}(1 - R_{\min})\}}{1 - R_{\min} \times \exp\{-\mathrm{Ntu}(1 - R_{\min})\}} \quad \text{for} \quad R_{\min} < 1 \tag{20a}$$

$$\varepsilon = \frac{\mathrm{Ntu}}{1 + \mathrm{Ntu}} \quad \text{for} \quad R_{\min} = 1 \tag{20b}$$

where, $R_{\min}$ is the minimum heat capacity ratio, for two fluids, it is calculated as $R_{\min} = \mathrm{Min}(1, R_1^{-1})$ according to the present formulation. It is important to note here that Ntu is based on the heat capacity of the hot fluid and total length of the heat exchanger (L = number of elements $\times$ L_e).

The test is selected to calculate the effectiveness for different values of the number of transfer units (Ntu) and fixed value of the heat capacity ratio parameter ($R_1 = 0.5$). The results of the subdomain collocation method and the Galerkin's method are listed in Tables I to IV.

Table II.
Comparison of the values of the effectiveness of the classical parallel-flow two-fluid heat exchanger ($H = 0$) using different number of elements (Galerkin's method) for $R_1 = 0.5$

			ε		
Ntu	Equation (19)	2 elements	4 elements	8 elements	16 elements
0.50	0.3518	0.3400	0.3452	0.3483	0.3500
1.00	0.5179	0.5000	0.5066	0.5116	0.5146
1.50	0.5964	0.5816	0.5854	0.5900	0.5929
3.00	0.6593	0.6600	0.6558	0.6568	0.6578
5.00	0.6663	0.6633	0.6662	0.6660	0.6661

Table III.
Comparison of the values of the effectiveness of the classical counter-flow two-fluid heat exchanger ($H = 0$) using different number of elements (Subdomain collocation method) for $R_1 = 0.5$

			ε		
Ntu	Equations (20a) and (20b)	2 elements	4 elements	8 elements	16 elements
0.5	0.3623	0.3626	0.3624	0.3623	0.3623
1.0	0.5647	0.5664	0.5651	0.5648	0.5648
1.5	0.6908	0.6944	0.6917	0.6910	0.6910
3.0	0.8744	0.8848	0.8769	0.8750	0.8746
5.0	0.9572	0.9726	0.9609	0.9581	0.9574

Table IV.
Comparison of the values of the effectiveness of the classical counter-flow two-fluid heat exchanger ($H = 0$) using different number of elements (Galerkin's method) for $R_1 = 0.5$

			ε		
Ntu	Equations (20a) and (20b)	2 elements	4 elements	8 elements	16 elements
0.50	0.3623	0.3572	0.3596	0.3609	0.3616
1.00	0.5647	0.5535	0.5587	0.5616	0.5631
1.50	0.6908	0.6755	0.6823	0.6863	0.6885
3.00	0.8744	0.8571	0.8636	0.8684	0.8713
5.00	0.9572	0.9467	0.9487	0.9522	0.9545

It can be seen from Tables I to IV that both the methods give acceptable accurate results even for two elements and the results of the subdomain collocation method are closer to the analytical results from that of the Galerkin's method. Therefore, the 16-element subdomain collocation method is used to analyze the performance of the counter flow three-fluid heat exchanger.

The results provide confidence to the accuracy of the present numerical method to study the performance of three-fluid heat exchanger. The governing parameters in this case are the heat transfer coefficient parameters between the hot fluid and the two cold fluids H, the heat capacity ratio parameters R_1 and R_2, the inlet temperature ratio parameter in addition to the Ntu. These parameters are defined in equation (8).

To study the effect of the heat transfer coefficient parameter H, the results are generated for constant values of $R_1 = R_2 = 1.0$ and same inlet temperatures for both cold streams $\Theta = 0$. Figure 2 shows the variation of the effectiveness with Ntu for different values of heat transfer coefficient H for both co-current and counter-current flow arrangements. The results of the classical two-fluid heat exchanger are also presented for comparison. It can be seen from Figure 2 that the results of $H = 0$ are identical to that of the two-fluid results for both parallel and counter flow arrangements. Increasing the values of H which means reducing the thermal resistance between the hot fluid and the second cold fluid leads to increase the heat transfer to the two cold fluids and hence increase the effectiveness of the heat exchanger. It can be seen also from Figure 2 that at high values of Ntu and H, the heat transfer from the hot fluid to cold fluids will reach the maximum possible heat transfer and the effectiveness

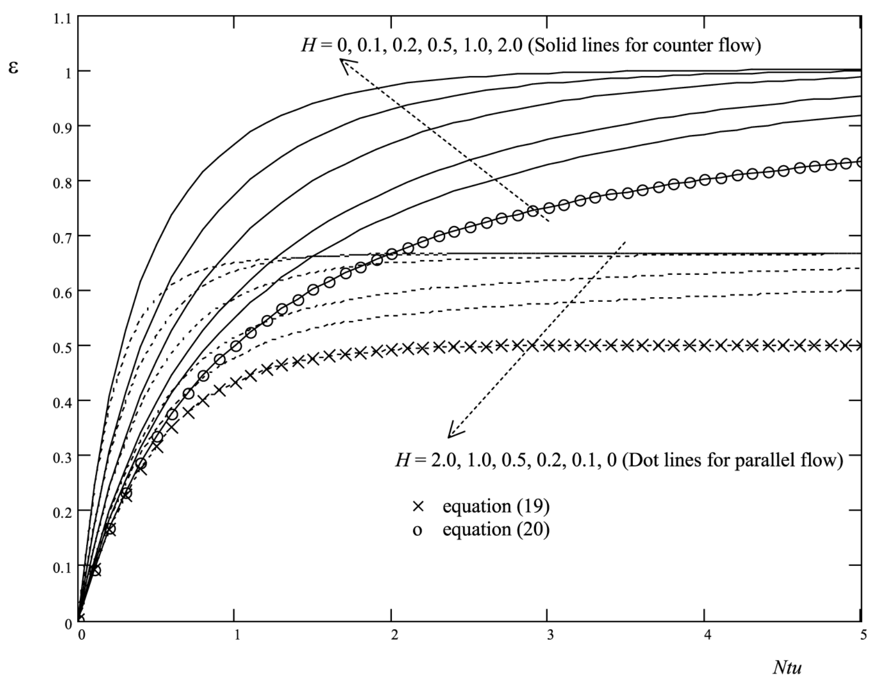

Figure 2.
Variation of the effectiveness with Ntu for different values of heat transfer coefficient ratio and fixed values of $R_1 = R_2 = 1.0$ and $\Theta = 0$

approaches unity for the counter-current flow arrangement. While for co-current flow arrangement, the effectiveness increases with increasing Ntu.

The effect of the heat capacity ratio parameter is studied where the heat transfer coefficient ratio is fixed $H = 1.0$ and equal inlet temperatures for both cold streams $\Theta = 0$. The variation of the effectiveness with Ntu for different values of heat capacity ratio R_2 are shown in Figure 3 for constant $R_1 = 1$ for both co-current and counter-current flow arrangements.

Figure 3 shows that maximum effectiveness is calculated for maximum value of Ntu and the case when $R_2 = 0$. Physically $R_2 = 0$ means that the second cold fluid has infinite heat capacity (equation (8)) which is not effected by the heat transfer to it and its temperature remains constant along the heat exchanger (ambient temperature). In this case, the heat is allowed to transfer to this cold fluid (ambient temperature) without changing its temperature.

Increasing the values of R_2 while keeping the other parameters constant, the effectiveness-Ntu variations is reduced which means that the heat transfer from the hot fluid to the cold fluids are reduced. This is due to reducing the heat capacity of the second cold fluid by increasing the values of R_2. Increasing the value of R_2 leads to increase in the temperature of the second cold fluid along the heat exchanger and hence reduce the heat transfer from the maximum possible value. For very large value of R_2 (when the heat capacity of the second cold fluid is negligible as compared with that of hot fluid), the heat exchanger can be considered as two-fluid heat exchanger. This fact is shown in Figure 3 by comparing the results of the classical two-fluid heat exchanger with that of three-fluids with $R_2 = 1{,}000$ for both co-current and counter-current flow arrangements.

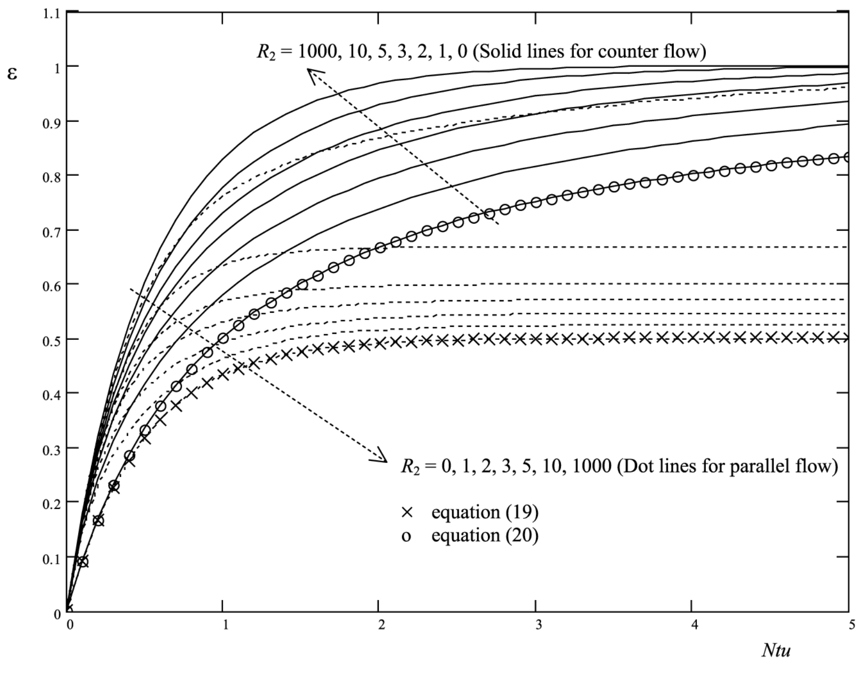

Figure 3.
Variation of the effectiveness with Ntu for different values of heat capacity ratio and fixed values of $H = 1.0$, $R_1 = 1.0$ and $\Theta = 0$

It can be seen also from Figures 2 and 3 that the effectiveness of the three-fluid counter flow heat exchanger is always higher than that of classical two-fluid counter flow heat exchanger due to the existence of one more medium to receive the heat from the hot fluid.

The effect of the third governing parameter, which is the inlet temperature ratio parameter Θ is studied with fixed values of other parameters $H = R_1 = R_2 = 1.0$. It is assumed that the two cold fluids can enter the three-fluid heat exchanger with different temperatures in both co-current and counter-current flow arrangements. The range of the inlet temperature ratio parameter Θ is considered from zero (equal cold fluids temperature at the inlet) to unity (the inlet temperatures of the hot fluid and the second cold fluid are equal).

Figure 4 shows the variation of the effectiveness with Ntu for different values of Θ and fixed values of $H = R_1 = R_2 = 1.0$ for both co-current and counter-current flow arrangements. It can be noticed from Figure 4 that for both the flow arrangements, maximum effectiveness can be obtained when the two cold fluids enter the heat exchanger at same temperature ($\Theta = 0$). Increasing the inlet temperature of the second cold fluid leads to reduction of the rate of heat transfer and hence also reduction of the effectiveness for both co-current and counter-current flow arrangements as shown in Figure 4.

It can be observed from Figures 2 to 4 that the effectiveness of the three-fluid heat exchangers can be improved by increasing the Ntu for some range of Ntu after which increasing Ntu has no significant effect on the effectiveness and it becomes approximately constant. For the co-current flow arrangement, it is clear from Figures 2-4 that this range of Ntu is small comparing with that for the counter-current flow arrangement.

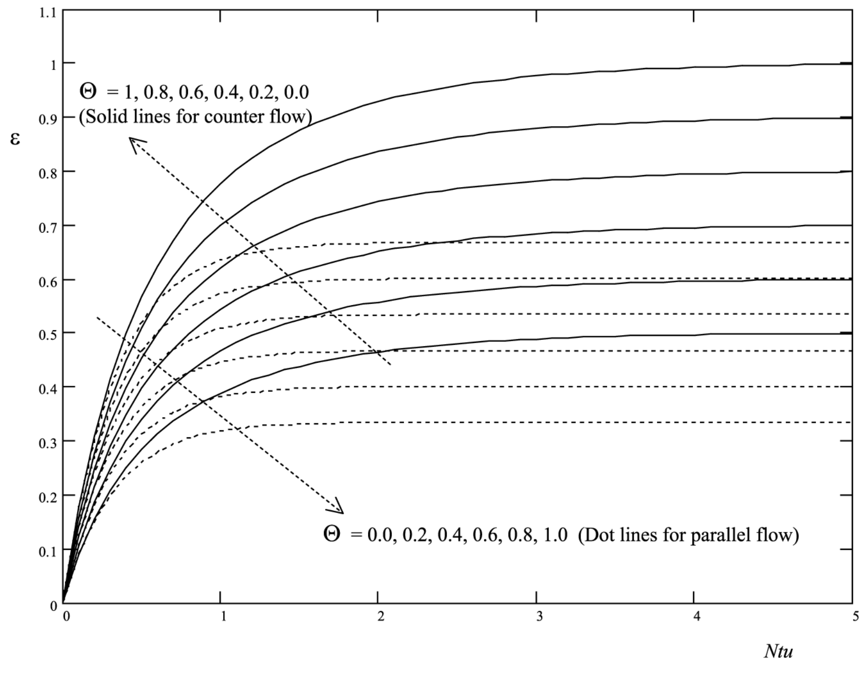

Figure 4.
Variation of the effectiveness with Ntu for different values of inlet temperature parameter and fixed values of $H = R_1 = R_2 = 1.0$

Finally, Figure 5 shows the effect of the heat capacity ratio on the heat exchanger effectiveness for five different values of Ntu and fixed heat transfer coefficient ratio $H = 1.0$ and $\Theta = 0$ for both the flow arrangements. For same heat capacity ratio of both cold channels ($R_1 = R_2$) and constant value of Ntu, the effectiveness decreases with the increase in the heat capacity ratio due to increasing the heat capacity of the hot fluid or decreasing the heat capacity of the cold fluid, where $R_1 = (\dot{m}c_\mathrm{p})_\mathrm{h}/(\dot{m}c_\mathrm{p})_{\mathrm{c}1}$.

For both the flow arrangements, the effectiveness shows a minimum when the heat capacity ratio equals 2, i.e. when the heat capacity of the hot fluid is two times that of the cold fluids. Figure 5 shows also that the effectiveness starts to increase with the increase in the heat capacity ratio more than the value of 2 for all values of Ntu. The reduction of the effectiveness is more pronounced for high values of Ntu. While for small values of Ntu, the effectiveness is almost constant with increasing the heat capacity ratio up to value of 2 after which the effectiveness starts to increase with increasing the heat capacity ratio. These results are similar to that presented by Rao *et al.* (2002) for the performance of a plate heat exchanger. It can be observed from Figure 5 that at small values of Ntu, the variation of the effectiveness for the co-current flow is very near to that for the counter-current flow. While at high values of Ntu, the large difference between the variations of the effectiveness for the co-current flow and counter-current flows occurs; counter-current flow yielding higher effectiveness for constant Ntu.

Figures 2-5 show that for fixed values of the governing parameters, the effectiveness of the counter-current flow arrangement is always higher than that for the co-current three-fluid heat exchangers.

5. Conclusions

In this study, the finite element method is used to study the performance of three-fluid heat exchanger. The governing parameters in this problem are the heat transfer coefficient parameter between the hot fluid and the two cold fluids H, the heat capacity

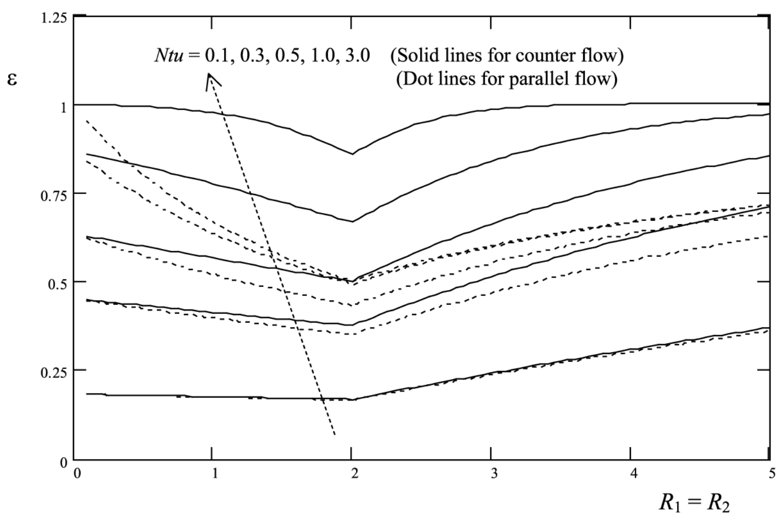

Figure 5.
Variation of the effectiveness with R for different values of Ntu and fixed values of $H = 1.0$ and $\Theta = 0$

ratio parameters R_1 and R_2, the inlet temperature ratio parameter Θ in addition to the Ntu.

It is found that, increasing the value of H and keeping the other parameters constant leads to increasing the effectiveness of the heat exchanger and at high values of Ntu and H, the heat transfer from the hot fluid to cold fluids will reach the maximum possible value. While increasing the value of R_2 by keeping the other parameters constant, the heat transfer from the hot fluid to the cold fluids is reduced. It is found also that the effectiveness of the three-fluid heat exchanger is always higher than that of classical two-fluid heat exchanger due to the existence of one more medium to receive the heat from the hot fluid.

For same heat capacity ratio of both cold channels ($R_1 = R_2$) and constant value of Ntu and Θ, the effectiveness decreasing with the increase in the heat capacity ratio. The effectiveness shows a minimum when the heat capacity ratio equals 2. The effectiveness starts to increase with the increase in the heat capacity ratio more than the value of 2 for all values of Ntu and both the flow arrangements.

It is observed that the effectiveness of the three-fluid heat exchangers can be improved by increasing the Ntu for some range of Ntu after which increasing Ntu has no significant effect on the effectiveness and it becomes approximately constant. The results show that for fixed values of the governing parameters, the effectiveness of the counter-current flow arrangement is always higher than that for the co-current three-fluid heat exchangers.

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Appendix

The details of the matrices in equation (16) for Subdomain collocation method are as follows:

$$[K] = \left[\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ \frac{\pm \mathrm{Ntu_e} R_1}{2} & \pm\left(1 - \frac{\mathrm{Ntu_e} R_1}{2}\right) & 0 \\ \frac{\pm \mathrm{Ntu_e} H R_2}{2} & 0 & \pm\left(1 - \frac{\mathrm{Ntu_e} H R_2}{2}\right) \\ \left(-1 + \frac{\mathrm{Ntu_e}}{2} + \frac{\mathrm{Ntu_e} H}{2}\right) & \frac{-\mathrm{Ntu_e}}{2} & \frac{-\mathrm{Ntu_e} H}{2} \end{matrix}\right.$$
$$\left.\begin{matrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ \frac{\pm \mathrm{Ntu_e} R_1}{2} & \pm\left(-1 - \frac{\mathrm{Ntu_e} R_1}{2}\right) & 0 \\ \frac{\pm \mathrm{Ntu_e} H R_2}{2} & 0 & \pm\left(-1 - \frac{\mathrm{Ntu_e} H R_2}{2}\right) \\ \left(1 + \frac{\mathrm{Ntu_e}}{2} + \frac{\mathrm{Ntu_e} H}{2}\right) & \frac{-\mathrm{Ntu_e}}{2} & \frac{-\mathrm{Ntu_e} H}{2} \end{matrix}\right]$$

where the positive sign for counter-current flow and negative sign for co-current flow and

$$[\theta'] = \begin{bmatrix} \theta'_{hi} \\ \theta'_{c1i} \\ \theta'_{c2i} \\ \theta'_{ho} \\ \theta'_{c1o} \\ \theta'_{c2o} \end{bmatrix} \quad \text{and} \quad [f] = \begin{bmatrix} BC1 \\ BC2 \\ BC3 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

where

BC1, 1 for the first element; BC2, 0 for the last element for counter flow or first element for parallel flow; BC3, Θ for the last element for counter flow or first element for parallel flow.

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